II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let in a triangle ABC $AB>AC$, and BE and CD be the bisectors of angles B and C of the triangle, cutting AC and AB in E and D, respectively. To prove $CD<BE$.

Through E draw EH parallel to CB. Draw HI parallel to AC, and HK parallel to DC, cutting AC in K. It is evident that $HE=HB$, and $HE=EK$; moreover, $\angle HIB=\angle ACB>\angle HBI$. Therefore, $BH>HI$, and hence $EK>HI$ or EC. The point K, therefore, lies on AC produced, and hence, the point D between A and H. Comparing the two triangles BHE and HEK, we see at once that $BE>HK$. But $CD<HK$, *a fortiori*. Therefore $CD<BE$.

Also solved by Henry Heaton, A. H. Holmes, Rev. J. H. Meyer, and G. B. M. Zerr.

286. Proposed by S. F. NORRIS, Baltimore City College, Baltimore. Md.

On the sides of a given triangle measure off equal distances from the extremities of the base, and at these points erect perpendiculars to the sides. Find the locus of the point of intersection of these perpendiculars. Solve by methods of analytic geometry.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ACB be the triangle, AB the base. Lay off $AE=BD=d$ on the two sides AC, BC, and erect perpendiculars at E and D cutting each other at H, and CB and AC at F and G, respectively. Then $CE=b-d$, $CF=(b-d)\sec C$.

Hence $\dfrac{x}{(b-d)\sec C}+\dfrac{y}{b-d}=1$, is the equation to EF..........(1).

Also $CD=a-d$, $CG=(a-d)\sec C$, and $\dfrac{x}{a-d}+\dfrac{y}{(a-d)\sec C}=1$, is the equation to DG..........(2). Now (1) and (2) may be written as follows:

$$x\cos C+y=b-d\ \text{........}\ (3),$$
$$x+y\cos C=a-d\ \text{.........}\ (4).$$

Eliminating d we get $x-y=\dfrac{a-b}{1-\cos C}=\frac{1}{2}(a-b)(\operatorname{cosec}\frac{1}{2}C)^2$, as the locus of H, the intersection required.

Also solved by G. W. Greenwood, Henry Heaton, A. H. Holmes, and J. Scheffer.

287. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill

Show that the points whose abscissae are 0, $a\sqrt{3}$, and $-a\sqrt{3}$ are points of inflexion on the locus $x^2y-a^2x+a^2y=0$.

Solution by the PROPOSER.

Let P be the point whose abscissa is $a\sqrt{3}$ and whose ordinate is therefore $\dfrac{a\sqrt{3}}{4}$. Let Q be any point on the curve. The coördinates of Q are, therefore,

$$a\sqrt{3}+r\cos\theta,\quad \frac{a\sqrt{3}}{4}+r\sin\theta,$$

where $PQ=r$, and the line PQ makes an angle θ with the x-axis. Hence, since Q lies on the curve, we have

$$2ar(\cos\theta+8\sin\theta)+\sqrt{3}\,ar^2\cos\theta(\cos\theta+8\sin\theta)+4r^3\cos^2\theta\sin\theta=0.$$

One value of r is zero for all values of θ; hence one branch of the curve passes through P. *Two* more values of r are zero, when $8\sin\theta+\cos\theta=0$. Hence P is a point of inflexion. In a similar manner we can show that the other points named are points of inflexion.

Also solved by A. H. Holmes, J. Scheffer, and G. B. M. Zerr.

PROBLEMS FOR SOLUTION.

ALGEBRA.

265. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Obtain the reduced cubic $4\theta^3-I\theta+J=0$ of the biquadratic $ax^4+4bx^3+6cx^2+4dx+e=0$.

266. Proposed by L. E. NEWCOMB, Los Gatos, Calif.

Find the nth term and the sum of n terms of the series $1+3+7+17+$.....

267. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

Express the trigonometric functions of x as infinite continued fractions.

CALCULUS.

221. Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, Ohio.

If a, b, c, ... represent all the prime numbers 2, 3, 5, ... prove that

$$\left(1+\frac{1}{a^2}\right)\left(1+\frac{1}{b^2}\right)\left(1+\frac{1}{c^2}\right)\ldots\ldots=\frac{15}{\pi^2}.$$

222. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Evaluate $\int_0^1(1+x^m)^n\log x\,dx$.

DIOPHANTINE ANALYSIS.

137. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Prove that all multiply perfect numbers of multiplicity $\varkappa$ having only n distinct primes are comprised in $n=2, 3, 4$.